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# Unfair competition in the information environment. Industrial information leakage

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## 1 Introduction

The purpose of the paper is the study of issues related to the R&D competition in the information era. It has been motivated by the modern methods of corporate espionage, so-called cyber espionage. That is when companies are using information technologies to obtain a competitive advantage unfairly. Specifically, the paper focuses on the information/knowledge leakage in the innovative industry sector.

The dynamic game model developed by Jennifer F. Reinganum is used as a base model. The basic model has been built to study issues related to industrial R&D. However, it has not been developed with unfair competition methods in mind. The paper attempts to present a modified version of a model and introduces an information leakage device, which allows the firm to steal the knowledge needed for the research to outrun its competitors.

The paper raised numerous questions and answered a few. The attempt to derive a tractable analytical solution for the new model appeared to be impossible. The tractable solution of this form is not feasible. The solution proposed is derived under heavy assumptions and cannot be used to develop adequate insights. However, the paper set up numerous future research vectors that will give birth to new projects.

The Paper is organised as follows: the first section introduces. Section 2 provides a research background and legacy. Section 3 sets up a modernised model. The derivation of the payoffs provided in Section 4. Section 5 attempts to derive a solution for the model set. Finally, Section 6 concludes.

## 2 Background

The term research and development with rivalry was introduced by Jennifer R. Reinganum in 1981 (Reinganum, 1981). Using this term she described a situation where firms are competing for the profits which accrue with the introduction of a particular innovation. She focused on the situation in which two firms compete to develop a new process or device. Since then literature is filled with various modifications and extensions of the basic idea. It is flexible enough to be enhanced and studied under different circumstances (e.g., availability of patent protection, inability to keep research findings secret, etc.).

The paper continues the legacy of Reinganum. It extends the basic case with perfect patent competition (the first firm to perfect the information is granted with a monopoly; imitation is forbidden). The framework extended with the assumption that firms are not able to keep their on-going R&D findings secret. Therefore, their knowledge could be leaked and used for the good of competitors. Consequently, the new framework has a wider range of opportunities for a resources allocation:

- conventional methods of information/knowledge acquisition and patent perfection (e.g. R&D);
- unfair methods of information/knowledge (e.g. cyber espionage (David J. Kappos)).

The technical aspects of unfair competition in the information environment will not be discussed in the paper. It is assumed to be any appropriate type of cyberattack or insider threat.

One of the most important features of the R&D races is the notion of uncertainty regarding the innovation timings. It means that every firm in a race should come up with a dynamic and adjustable planning being able to correct their resource allocation policy at any point of time of the planning horizon. The uncertainty issues address the dynamic nature of competition, contingency planning and R&D itself.

The model presented in the paper uses combined game theoretic (Dockner, 2000) and decision theoretic approach (Kamien  Schwartz, 1991). It considers the rivals as strategic agents that can derive their own best responses to rivals‘ given strategies, rivals‘ best responses and a Nash equilibrium. The strategy is modelled as a function of time. The resulting technique implies differential games or games played in a continuous time.

# 3 Setting

The paper assumes that are only two identical firms (players) that are competing in a race for a particular innovation (e.g., an invention or process). The $n - player$ generalisation will be done in the future research.

Firstly it is necessary to come up with basic definitions. The paper uses similar definition setting to that presented in the Reinganums work (Reinganum, 1981).

*Definition 1.* Player $i$ succeeds if $i$ perfects the innovation.

*Definition 2.* Player $i$ wins if $i$ succeeds before the rival (player $j$) does.

The winner receives the patent and a monopolistic exploitation rights, which, in turn, generate the rent in the present values. The looser gains nothing.

In our case, we assume that firms acquire only the knowledge relevant to the innovation. As mentioned above there are two sources of knowledge acquisition and each firm chooses their resource allocation plan. For the sake of simplicity, knowledge acquisition is deterministically defined for both sources:

- investment in R&D directly increases the stock of knowledge;
- investment in the information leakage directly and steadily increases the stock of knowledge on account of competitors knowledge stock;

Information leakage modelled with simplicity in mind leaves room for the future research, where it can be determined stochastically addressing the uncertainty of the crime. There is also no allowance for knowledge stock decrease. Once obtained, knowledge maintains within the stock. Innovation modelled stochastically addressing the uncertainty mentioned above. Therefore, the time of success is a random variable, which can be indirectly influenced by a firm through increase in its stock of relevant knowledge.

(S1) $T$ - calendar day of Doomsday. The paper considers the doomsday as a time at which innovation becomes archaic, so firms are no longer interested in the patent. It can also be regarded as a date after which innovation is considered infeasible, hence firms drop the innovation process;

(S2) $r$ - common discount rate;

(S3) $P(t)$ - present value if the patent obtained at random time $t$. For the sake of simplicity, it is assumed that $P(t) = P$, where $P$ is a positive constant. It remains unchanged during the whole race period;

(S4) $t_i$ - firms $i$ random time of success;

(S5) $u_i(t)$ - is the rate at which firm $i$ **fairly** acquires knowledge at time $t$. It is the amount of relevant knowledge that firm chooses to obtain at time t. The paper assumes that $u_i(t) \in [0, B], B > 0$. Therefore, there exists a limit for the knowledge acquisition in one instance;

(S6) $\gamma_i z_j$ - is the rate at which firm $i$ **unfairly** acquires knowledge at time $t$. $\gamma_i$ is a share of rival's knowledge that firm $i$ chooses to steal at time $t$. The nature of the variable imposes physical limits on the amount of knowledge that can be stolen (e.g. it is not possible to steal more knowledge that the rival has). The paper assumes that $\gamma \in [0, 1]$;

(S7) $e^{-rt}c_i(u_i)$ - discounted cost of additional knowledge $u_i$ **fairly** acquired by firm $i$ at time $t$. The paper proposes the quadratic cost function for the sake of simplicity: $c_i(u_i) = 1/2(\mu_i)^2$;

(S8) $e^{-rt}c_i(\gamma_i)$ - discounted cost of additional knowledge $\mu_j$ **unfairly** acquired by firm $i$ at time $t$. Similarly, the paper introduces the quadratic cost function for the information leakage: $c_i(\mu_j) = 1/2(\gamma_i)^2$;

(S9) $z_i(t)$ - firms $i$ knowledge stock at the time $t$;

(S10) $\dot{z}_i(t) = u_i(t) + \gamma_i(t)_{z_j}$ - overall knowledge acquisition rate at time $t$. Includes fairly and unfairly obtained knowledge at the time instance.

(S11) $F_i(z)$ - the likelihood of firms $i$ success with knowledge stock of $z$ at $t$ or before *it*. The paper specifies that $F_i(z) = 1 - e^{-\lambda z_i(t)}$. The use of exponential distribution is convenient due to the memorylessness property. It means that a waiting time until certain event does not depend on how much time has already elapsed (Feller, 1957);

(S12)

(S13) $Pr(t_i \in (t, t+\delta], t_i > t) = \frac{dF_i}{1-dF_i} = \frac{(-)(-\lambda)(u_i(t)+\gamma_i(t)z_j(t))e^{-\lambda z_i}}{e^{-\lambda z_i}} = \lambda(u_i(t) + \gamma_i(t)z_j(t))$ - conditional probability that firm $i$ will succeed in the next instant, given that is has not already done so;

(S14) Vectors are denoted using unsubscripted letter (e.g. $z = (z1, z2)$).

$u_i$, $\gamma_i$ are considered as control functions for the dynamic optimal control problem.

*Definition 3.* Control functions for the player $i$ are functions, which take values in the control space $U_i = [0, B]$ for all $t \in [O, T]$ (for $u_i$) and in the $\Gamma_i = [0, 1]$ for all $t \in [O, T]$ for $(\gamma_i)$.

# 4 Constructing payoffs

From this point, we will consider the game from the player's $i$ point of view. The generalisation for the player $j$ is straight forward.

Player's 1 payoff consist of two terms:

(P1) Firm 1 receives the amount $P$ (in present value terms) if it succeeds at $t$ and if firm 2 has not yet succeeded. Hence, the first term of payoff:

$$(1-F_2)f_1P = Pe^{-\lambda z_2}\lambda \dot{z}_i(t)e^{-\lambda z_1} = Pe^{-\lambda(z_1+z_2)}\lambda(u_i(t)+\gamma_i z_j(t))$$

(P2) Development (knowledge acquisition) costs will accompany research activity as long as none of the players has succeeded. Hence, the second term of payoff:

$$(1-F_1)(1-F_2) = \frac{1}{2}e^{-\lambda(z_1+z_2)}(u_i^2+\gamma_i^2)$$

Putting terms together and discounting the dynamic costs at rate $r$, the firm i's payoff for any strategy set $(u_1, u_2, \gamma_1, \gamma_2)$ can be written:

$$J^i(u_1,u_2,\gamma_1,\gamma_2) = \int_0^T\{Pe^{-\lambda(z_1+z_2)}\lambda(u_i+\gamma_i z_j) - \tfrac{1}{2}e^{-\lambda(z_1+z_2)}(u_i^2+\gamma_i^2)\}dt$$

By integrating the first term by parts, we can obtain the terminal condition and deal with the following payoff:

$$\begin{aligned} J^1(u_1,u_2,\gamma_1,\gamma_2) &= \int_0^T\{P\lambda(u_j+\gamma_j z_i)e^{-\lambda z_j}(1-e^{-\lambda z_i}) \\ &\quad -\frac{1}{2}e^{-\lambda(z_1+z_2)}(u_i^2+\gamma_i^2 z_j)\}dt \\ &\quad +Pe^{-\lambda z_j(T)}(1-e^{-\lambda z_i(T)}) \end{aligned}$$

# 5 The Noncooperative Game With Information Leakage Device

*Definition 4.* Nash equilibrium is a solution concept of a non-cooperative game involving two or more players in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy (Osborne  Rubinstein, 1994).

*Definition 5.* A pure strategy for the firm $i$ are elements $u_i$ and $\gamma_i$ that belong to strategy spaces $U_i$ and $\Gamma_i$ respectively. Pure strategies for $i$ are functions of time and relevant knowledge stock. The literature recalls this type of strategy as a closed-loop strategy (Dockner, 2000).

This section attempts to derive the Noncooperation Nash equilibrium state. For this purpose, the paper introduces the value function.

A strategy set $(u_1^*, u_2^*, \gamma_1^*, \gamma_2^*)$ is a Nash equilibrium IFF:

1. $J^1(u_1^*, u_2^*, \gamma_1^*, \gamma_2^*) \geq J^2(u_1, u_2^*, \gamma_1, \gamma_2^*)$ for all $u_1, \gamma_1$;
2. $J^2(u_1^*, u_2^*, \gamma_1^*, \gamma_2^*) \geq J^2(u_1^*, u_2, \gamma_1^*, \gamma_2)$ for all $u_2, \gamma_2$.

The paper assumes that Nash equilibrium exists and attempts to characterise it. The associated value function is presented below:

$$\begin{aligned} V^1(u_1, u_2, \gamma_1, \gamma_2) &= \int_s^T \{P\lambda(u_j + \gamma_j z_i)e^{-\lambda z_j}(1 - e^{-\lambda z_i}) \\ &\quad -\frac{1}{2}e^{-\lambda(z_1+z_2)}(u_i^2 + \gamma_i^2 z_j)\}dt \\ &\quad +Pe^{-\lambda z_j(T)}(1 - e^{-\lambda z_i(T)}) \end{aligned}$$

where $y = z(s)$

Strategy set $(u_1^*, u_2^*, \gamma_1^*, \gamma_2^*)$ must satisfy a system of $Hamilton-Jacobi-Bellman$ (HJB) equations (Friedman, 2006):

$$\begin{aligned} &V_t^i + max[V_{z_i}^i(u_i + \gamma_i z_j) + V_{z_j}^i(u_j^* + \gamma_j^* z_i) \\ &+P\lambda(u_j^* + \gamma_j^* z_i)e^{-\lambda z_j}(1 - e^{-\lambda z_i}) \\ &-1/2e^{-rt}e^{-\lambda(z_1+z_2)}(u_i^2 + \gamma_i^2)] = 0 \end{aligned}$$

Where $u_i(t,z) \in [0,B]$, $\gamma_i(t,z) \in [0,1]$ and terminal condition:

$$V^i(T, z(T)) = Pe^{-\lambda z_j(T)} - Pe^{-\lambda(z_1(T)+z_2(T))}$$

Performing the optimisation presented above it is possible to obtain Nash equilibrium candidates. As the left-hand side of HJB is assumed to be strictly concave in $u_1$ and $\gamma_1$, the necessary maximum conditions are sufficient as well.

Proceeding with optimization for $u_i$ yields:

(M1) $V_{z_i}^i - e^{-rt}e^{-\lambda(z_1+z_2)}u_i > 0 \ \forall u_i \in [0,B] \Rightarrow u_i^* = B$

(M2) $V_{z_i}^i - e^{-rt}e^{-\lambda(z_1+z_2)}u_i = 0 \Rightarrow u_i^* = V_{z_i}^i e^{rt}e^{\lambda(z_1+z_2)}$

(M3) $V^i_{z_i} - e^{-rt}e^{-\lambda(z_1+z_2)}u_i < 0 \ \forall u_i \in [0, B] \Rightarrow u^*_i = 0$

Optimization for $\gamma_i$ yields:

(V1) $V^i_{z_i}z_j - e^{-rt}e^{-\lambda(z_1+z_2)}\gamma_i > 0 \ \forall \gamma_i \in [0, 1] \Rightarrow \gamma^*_i = 1$

(V2) $V^i_{z_i}z_j - e^{-rt}e^{-\lambda(z_1+z_2)}\gamma_i = 0 \Rightarrow \gamma^*_i = V^i_{z_i}z_je^{rt}e^{\lambda(z_1+z_2)}$

(V3) $V^i_{z_i}z_j - e^{-rt}e^{-\lambda(z_1+z_2)}\gamma_i < 0 \ \forall \gamma_i \in [0, 1] \Rightarrow \gamma^*_i = 0$

The interpretation of conditions is quite straightforward. If the additional expected profit generated by investment in the R&D ($V^i_{zi}$) or by the cyberattack ($V^i_{zi}z_j$) exceeds the discounted marginal costs $e^{-rt}e^{-\lambda(z_1+z_2)}u_i$ and $e^{-rt}e^{-\lambda(z_1+z_2)}\gamma_i$ at the maximum rates $B$ (for $u_i$) and 1 (for $\gamma_i$) then the firm making an all-effort push to finish the project: $u^*_i = B$ and $\gamma^*_i = 1$. In the situation, where additional expected profit generated by any source of knowledge acquisition is exceeded by the discounted marginal costs, we have the opposite corner solution: $u^*_i = 0$ and $\gamma^*_i = 0$.

However, for the derivation of the Nash equilibrium, the paper looks into the interior solutions for the problem. In this situation, the additional expected profit is exactly balanced with the discounted marginal costs. Obviously, we cannot know an exact functional form of the Value function $V$. Therefore the exact relationships between the function and the independent parameters (e.g., $\lambda$, $r$, $T$) are not feasible. It is still possible to substitute the expressions for $u_i$, $u_j$, $\gamma_i$ and $\gamma_j$ back to the HJB equation and make an attempt to solve it for the value functions $V_1$ and $V_2$ in order to determine Nash equilibrium strategies.

The paper interested in the following expressions to substitute :

(Q1) $u^*_i = V^i_{z_i}e^{rt}e^{\lambda(z_1+z_2)}$

(Q2) $u^*_j = V^j_{z_j}e^{rt}e^{\lambda(z_1+z_2)}$

(Q3) $\gamma^*_i = V^i_{z_i}z_je^{rt}e^{\lambda(z_1+z_2)}$

(Q4) $\gamma^*_j = V^j_{z_j}z_je^{rt}e^{\lambda(z_1+z_2)}$

Processing with the substitution and simplification yields the following system of HJB equations:

$$V^i_t + (\frac{1}{2} + \frac{1}{2}z^2_j)(V^i_{z_i})^2e^{rt}e^{\lambda(z1+z2)} + (1 + z^2_i)V^i_{z_j}V^j_{z_i}e^{rt}e^{\lambda(z1+z2)}$$
$$+(1 + z^2_i)V^j_{z_j}P\lambda e^{rt}(e^{\lambda z_i} - 1) = 0$$

With terminal conditions:

$$V^i(T, z(T)) = Pe^{-\lambda z_j(T)} - Pe^{-\lambda(z_1(T)+z_2(T)))}$$

Mentioned equation could be solved using the Method of undetermined coefficients (Boyce DiPrima, 1986). The terminal condition could be used to propose a form of the solution to the system:

$$V^i(T, z(T)) = k(t)e^{-\lambda z_j} + b(t)e^{-\lambda(z_1+z_2)}$$

It will be also convenient to derive the marginal values for the presented solution:

$$V^1_{z_1} = V^2_{z_2} = -b(t)\lambda e^{-\lambda(z_1+z_2)}$$

$$V^1_{z_2} = V^2_{z_1} = -b(t)\lambda e^{-\lambda(z_1+z_2)} - k(t)e^{-\lambda z_j}$$

In regard to the form of the marginal values, it becomes obvious that the HJB system can be reduced into two independent partial differential equations (PDE's). The paper will make an attempt to solve them separately. Substituting the derived solution form and its marginal value's into the HJB system yields:

$$e^{-\lambda(z1+z2)}(\dot{b}(t) + (\frac{3}{2} + \frac{1}{2}z_j^2 + z_i^2)b^2(t)\lambda^2 e^{rt} + (1 + z_i^2)b(t)P\lambda^2 e^{rt})$$

$+ \mathrm{e}^{-\lambda z_j}(\dot{k}(t) + (1 + z_i^2)b(t)k(t)\lambda^2 e^{rt} - (1 + z_i^2)b(t)P\lambda^2 e^{rt} = 0$

It is obvious that expressions $e^{-\lambda(z1+z2)}$ and $e^{-\lambda z_j}$ are always positive so that the HJB system can be decomposed into two separate PDEs with terminal conditions:

$$\dot{k}(t) + (1 + z_i^2)b(t)k(t)\lambda^2 e^{rt} - (1 + z_i^2)b(t)P\lambda^2 e^{rt},\ k(T) = P$$

$$\dot{b}(t) + (\frac{3}{2} + \frac{1}{2}z_j^2 + z_i^2)b^2(t)\lambda^2 e^{rt} + (1 + z_i^2)b(t)P\lambda^2 e^{rt},\ b(T) = -P$$

The first nonhomogeneous linear ordinary differential equation is straightforward to solve. It has a unique solution $k(t, P) = P$ that satisfies the terminal condition $k(T) = P \ \ \forall b(t, -P)$.

The problem arises with the second equation. Since $z_i$ and $z_j$ are also functions of time, the right form for the equation is:

$$\dot{b}(t) + (\frac{3}{2} + \frac{1}{2}z_j^2(t) + z_i^2(t))b^2(t)\lambda^2 e^{rt} + (1 + z_i^2(t))b(t)P\lambda^2 e^{rt},\ b(T) = -P$$

The tractable analytical solution of the mentioned form of the equation is not feasible. The paper came to the dead end in its attempt to derive an appropriate form of the Nash equilibrium. It is still, however, possible to obtain the numerical solution for the model. It will be done in the future research.

For now, it is still possible to analyse and try to characterise a Nash equilibrium assuming that functions $z_i$ and $z_j$ are considered as constant coefficients. The assumption is very heavy and makes the interpretation doubtful. However, it is still interesting to observe the behaviour of value function under the assumption provided. The derived Nash equilibrium characteristic will be compared with the Nash equilibrium derived by Reinganum (Reinganum, 1982).

Under the assumption provided the equation has a unique solution:

$$b(t,-P) = \frac{-C_1}{P\lambda^2 C_2(C_2 - exp(C_2 r^{-1} P\lambda^2(e^{rt} - e^{rT}))C_1)}$$

Where: $C_1 = \frac{3}{2} + \frac{1}{2}z_j^2 + z_i^2$ and $C_2 = 1 + z_i^2$.

In turn, $z_1$ and $z_2$ in $C_1, C_2$ assumed to be positive constants. Mathematically it could be explained by the assumption that $z_1$ and $z_2$ are extremely big variables, and knowledge acquisition has an insignificant impact on its values (hence it stays constant through the time). Of course, it is not the case in the model presented.

The sign of the derived $b(t,-P)$ is ambiguous and heavily depends on the values of $T$, $r$ and $\lambda$. Of course, we can suggest that sign of $b(t,-P)$ is possible. Then, we conclude that $C_2 > exp(C_2 r^{-1} P\lambda^2(e^{rt} - e^{rT}))C_1$. It is not a straightforward suggestion, but it does not contradict with the original basic model presented before.

Having $b(t,-P)$ and $k(t,P)$ coefficients we can now obtain the value function for Nash rivals:

$$V^i(t,z) = \frac{-C_1 e^{-\lambda(z1+z2)}}{P\lambda^2 C_2(C_2 - exp(C_2 r^{-1} P\lambda^2(e^{rt} - e^{rT}))C_1)} + Pe^{-\lambda z_j}$$

Where: $C_1 = \frac{3}{2} + \frac{1}{2}z_j^2 + z_i^2$ and $C_2 = 1 + z_i^2$.

We observe that the second term of the value function has not changed at all due to the independence of $k(t,P)$ coefficient. The first term changed significantly. The sign between the first term from the original function and the modified one is unclear.

The marginal benefit to the firm $i$ increasing its own knowledge stock is:

$$V^i_{z_i}(t,z) = \frac{C_1 \lambda e^{-\lambda(z1+z2)}}{P\lambda^2 C_2(C_2 - exp(C_2 r^{-1} P\lambda^2(e^{rt} - e^{rT}))C_1)} > 0$$

Where: $C_1 = \frac{3}{2} + \frac{1}{2}z_j^2 + z_i^2$ and $C_2 = 1 + z_i^2$.

Using the suggestion proposed above it is straightforward that this term is positive, which does not contradict with logic.

The increase in the rival knowledge has an ambiguous effect on the firm's $i$ profits. Thus:

$$V^i_{z_j}(t,z) = \frac{C_1\lambda e^{-\lambda(z1+z2)} - (P^2\lambda^3 C_2(C_2 - exp(C_2 r^{-1}P\lambda^2(e^{rt} - e^{rT}))C_1)}{P\lambda^2 C_2(C_2 - exp(C_2 r^{-1}P\lambda^2(e^{rt} - e^{rT}))C_1)}$$

Where: $C_1 = \frac{3}{2} + \frac{1}{2}z_j^2 + z_i^2$ and $C_2 = 1 + z_i^2$.

The sign of that term heavily depends on the cyberattack effort and available leakage methods. Therefore, it could have a positive effect if the firm has enough resources (both financial and technical) to leak opponents' knowledge, and negative otherwise.

Now the paper suggests the Nash equilibrium in pure strategies:

$$u_i^* = V^i_{z_i} e^{rt} e^{\lambda(z_1+z_2)} = \frac{C_1\lambda e^{rt}}{P\lambda^2 C_2(C_2 - exp(C_2 r^{-1}P\lambda^2(e^{rt} - e^{rT}))C_1)}$$

$$\gamma_i^* = V^i_{z_i} z_j e^{rt} e^{\lambda(z_1+z_2)} = \frac{C_1\lambda z_j e^{rt}}{P\lambda^2 C_2(C_2 - exp(C_2 r^{-1}P\lambda^2(e^{rt} - e^{rT}))C_1)}$$

As solution $(V^1(t,z), V^2(t,z))$ presented to the HJB system and its terminal conditions is continuous and differentiable, then $u_i^*$ and $\gamma_i^*$ satisfies the hypothesis of sufficiency Theorem 18.1 (Bernstein, 1950). Consequently, presented $u_i^*$ and $\gamma_i^*$ are the Nash equilibrium in the pure strategies.

Further study of the presented solution will not lead to the adequate conclusions due to the assumption presented.

# 6 Conclusion

The modified model for the rivals engaged in R&D is presented in the paper. The model includes not only the representation of the conventional knowledge acquisition but also the representation of unfair information/knowledge acquisition possibilities. Cyber-attack device has not been specified in the paper in details. However, it could be considered as any appropriate type of cyber-attack that leads to information leakage, or insider threat. The main purpose of the model is to study the corporative agents behaviour in the information environment and to come up with useful insights into corporative cyber-crime regulations.

The paper offers more questions than answers. There are two key issues to elaborate further:

Firstly, the analytical solution for the setting provided in the model is appeared to be infeasible. The solution presented is derived under heavy assumptions and cannot be used to make adequate conclusions. Due to the infeasibility of the analytical solution, it becomes obvious that the model should be solved numerically. The numerical solution for the model will be presented in the future research.

Secondly, the proposed form for the knowledge acquisition through information leakage is oversimplified and does not correctly reflect real-life processes. This has been done deliberately only for the initial model. It could be also modified in the future research as soon as the basic modified model will be solved.

Other issues to be addressed include: game asymmetry, generalisation for $n - players$,the introduction of defensive cyber-security capabilities and others.

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