UNIVERSITY OF OXFORD

CENTRE *for* DOCTORAL TRAINING *in* CYBER SECURITY

CDT Technical Paper

02/18

Multi-stage contest with cheating (working paper)

Oleh Stupak

Multi-stage contest with cheating [DRAFT]

Oleh Stupak

October 29, 2018

1 Introduction

Competition arises when parties strive for a goal the benefit of which cannot be shared. History of humanity itself is a history of conflicts, wars and conquests. We compete for territory and scarce resources, awards and social recognition, niche, profits, mates, etc.

Contests are surrounded by numerous issues and are under the close attention of economists. This paper discusses the incentives behind fraudulent output-enhancing activities better known as cheating.

Cheating is receiving an award or an ability to find a shortcut from an unpleasant situation by dishonest means. Cheating incentives in tournament-like situations may often arise from asymmetries in contestants' relative positions. This asymmetry can occur even in the situation in which players were initially identically endowed. As an example, consider two companies competing in an R&D race for a single patent, which have identical research budgets and similar researchers. Beyond the initial time point, it is very unlikely that companies' will ever be at the same stage of the research. This phenomenon appears due to the random processes involved in the innovation process: one contestant can just be more lucky.

The paper analyses cheating incentives that arise from the difference in relative position that occurred during the tournament among initially similar competitors. If the gap between contestants is sufficiently large, it will be more tempting for an underdog to cheat. This misbehaviour simply increases her chances of success and, therefore, her expected outcome. In the R&D case, if a luckier research company will be significantly ahead, it will create a strong incentive for a tailing opponent to cheat. Cheating could take the form of industrial espionage, insider threat or just some cyberattack which leads to a data breach.

It is easy to find examples like mentioned above in a modern news feed: this month's technological giant Super Micro's supply chain infiltration, which created a backdoor for Chinese adversaries in some of their server equipment - is just a fresh header that highlights protracted cyber arms race between United States and China (Reuters, 2018). Industrial espionage and similar dishonest behaviour constitute an enormous loss for global economy and undermine the nature of competition.

It is a known fact that competition rewards are seldom based on the absolute performance of participants. More commonly contesters are being rewarded based on their relative performance: only the fastest runner wins a gold medal during the Olympics, the company with the largest market share exploits monopolistic benefits, only scientists with most influential research can be internationally recognised.

Of course, the decision on cheating heavily relies on the costs that it bears. Cheating rarely comes for free. There is always the possibility of being caught which could lead to serious penalties (e.g. court fine for the thief of intellectual property), as well as possible reputational losses due to the violation of integrity or even emotional costs (e.g. pangs of conscience).

While the latter is almost impossible to quantify, the other cheating costs are a whole different story. They can be measured and even regulated by the tournament designer: fine for stealing an intellectual property is determined by law.

The paper attempts to discover the relationships between contestant's strategic decision to cheat, her effort, her relative advantage in the contest, cheating costs and uncertainty generated by a random factor, or luck.

To establish these relationships the paper uses a modified version of rank-order tournament initially introduced by Lazear and Rosen (1981), followed by Nalebuff and Stiglitz (1983).

In this article, I consider a game in which two homogeneous players compete over two-stage tournament by making a strategical decision. This can be thought as R&D contest for a patent in which players choose their investment levels at the initial stage and then decide on whether to cheat or not after they observe their relative positions. The precise description of the stages could be found below in the model section. The paper assumes that cheating decision leads to zeroing of relative advantage value and, therefore, equalizing the probability of their success. The focus of the article is on how equilibrium effort levels and the cheating decision is affected by varying the cheating costs and uncertainty.

Consistent with common sense, the paper discovers that underdog player has a strong incentive to cheat if her opponent is sufficiently ahead and cheating costs are reasonably small. Base model identified that costs of cheating have a positive impact on players' effort levels (the result may be changed. Meaning, for example, that the higher the penalty set by regulator, the more effort will be exerted by participants in the fair play. The paper also discovered that uncertainty has a negative influence on players' effort levels, which cam be explained by the notion of risk. Highly defused shocks imply that the choice of high effort can be easily offset by unlucky random event. Therefore, there is a high probability that this high effort will just lead to extra costs and will decrease the overall payoff.

The paper proceeds as follows: Section 2 overviews related literature. Section 3 presents a multi-stage contest model with cheating. The solution to the model is presented in Section 4. Section 5 discusses the model's limitations and suggests vectors for further development.

2 Brief literature review

Almost four decades ago Lazear and Rosen (1981) have published their famous paper "Rank-Order Tournaments as Optimum Labor Contracts". The original article contributed to the subfield of personnel economics and invented a new field: tournament theory. The theory described situation in which wage differentiation is based on relative differences among individuals rather than marginal productivity. It was followed by numerous papers on comparative analyses of individual contract and rank-order tournament compensation schemes and analysed related issues.

Nalebuff and Stiglitz (1983) who derived similar to original results, offered new modifications of the model and contributed to the multiagent, single-principal model. Hart (1983) analysed optimal contract designs and market competition in relation to managerial slack. Holmstrom (1982) presented multiagent partnership moral hazard framework and analysed free riding and competition issues. Green and Stokey (2016) investigated the influence of shocks on principal's choice of the payment scheme and, similarly to Lazear and Rosen, authors concluded that tournament reward schemes dominate in environments with significant uncertainty. O'Keeffe et al. (1984) were the first to step back from comparative analysis of compensation schemes. Authors have analysed the abstract rank-order tournament over indivisible good and focused on optimal contest conduction, its properties (e.g. effort monitoring precision, and prize spread) and the issues of fair play and asymmetric players. It was one of the first papers that covered misbehaviour in rank-order tournament setting.

Tournament theory found its implementation in various spheres of human activity: from human resource practice to professional sports and gaming. It was also applied to writing, art, law and, of course R&D (even competing research companies are relatively similar, only marginally better company will get a patent). Individuals always seek the easiest way to achieve their goal, and often this way appears to be out of notion of fair play.

Berentsen and Lengwiler (2005) argued that fraudulent accounting in companies (to embellish their financial status) and the use of doping in sports (to enhance performance) are relatively similar from game-theoretic perspective. Authors studied cheating in corporate and sports environment. They investigated the continuous time heterogeneous players framework where players can improve their performance via using performance enchantment drugs (that they referenced as the doping game). They discovered cyclical dynamics of cheating and the special cases where high-ability contestants are more likely to cheat than the low-ability ones.

The doping game was also studied by Kräkel (2007). He analysed two asymmetric players dynamics in tournament with cheating. The paper discovered three determinants of player's cheating decision: the likelihood effect (cheating increases the winning probability), the cost effect (cheating influence on the effort costs) and the windfall-profit (base wages) effect (the decrease in base-salary due to the possibility of getting caught). They also compared underdog's and leader's incentives to cheat, impact of player's heterogeneity on overall output and decisions, and schemes of doping prevention.

Curry and Mongrain (2000) implemented traditional enforcement model's insights into rank-order tournament setting. Authors claim that prize structure plays a crucial role in determining incentives of cheating. They consider a way in which the reward structure could be manipulated to reduce the costs of traditional deterrence instrument – monitoring.

My article is closely related to the papers of Gilpatric (2011) and Stowe and Gilpatric (2010) on cheating, enforcement and audit in contests. In his 2010 paper the author analysed the incentives behind prohibited behaviour in rank-order tournaments. In his single period framework analysis, he shows how the prize structure, uncertainty, monitoring efficiency, number of contestants and associated penalty impact cheating decision. Gilpatric also discovered that there are some circumstances in which greater cheating enforcing can reduce cheating, while also decrease productive effort.

Stowe and Gilpatric (2010) elaborate ideas of the latter paper but shift the focus entirely to the cheating decision itself and monitoring. The new framework could be considered as a final stage of tournament, where effort levels are already known, and the players decide only on cheating. The paper analyses different monitoring schemes and discovers that the audit based on players' relative positions is significantly more efficient in achieving a full deterrence than the scheme which monitors players with equal probability.

The idea of multistage contest with cheating described above has been developed further in my paper. Here I investigate the four-stage contest, where players have to decide on their effort levels and cheating decisions. Contrary to Gilpatric's setting, mine allows to investigate the influence of cheating costs (e.g. enforcement or penalty) on the productive effort in the multistage environment with multiple shocks. My setting also allows for the study of heterogeneous cheating penalties and shocks, which will be conducted later as a model's modification.

It also worth mentioning that my paper was influenced by Denter and Sisak (2015), Denter and Sisak (2016) and Siegel (2014) who analysed head starts in multistage contests under symmetric and asymmetric assumptions. In their papers authors discover that awarding a small head start to one of the players whenever players interact over multiple stages is optimal even with symmetric players. However, authors did not consider the possibility of cheating in their settings.

3 Model

Two agents, $i = j, k$, compete for a prize of common value $w = 1$ over four stages, $t = 1, 2, 3, 4$.

1. In the **first stage** each agent may exert effort x_i to increase her chances of receiving the prize at cost $c(x_i)$. The paper assumes $c'(x_i) > 0$ for all $x_i > 0$, $c'(0) = 0$, and $c''(x_i) > 0$;

2. In the **second stage** random shock $\epsilon \sim F$ is realised. F is assumed to be some symmetric distribution with infinite support. At the the end of the stage each player is able to observe its relative position (the value of d_i): player j has advantage of $d_j = x_j - x_k + \epsilon$, player k correspondingly $d_k = x_k - x_j - \epsilon$;

3. In the **third stage** players can pay v to cheat, which resets its rivals' advantage to zero. Let $\tilde{d}_i$ denote the relative position of the player at the end of $t = 3$. For example, $\tilde{d}_i = d_i$ if neither of the players cheat; $\hat{d}_i = 0$ if one of the players cheat;

4. In the **forth stage** a new random shock $\hat{\epsilon} \sim F$ is independently realised. Player j then wins the prize if $\tilde{d}_j + \hat{\epsilon} \geq 0$ and player k wins if $\tilde{d}_k - \hat{\epsilon} \geq 0$.

4 Solution

The game is being solved by backward induction beginning at the stage three.

4.1 t=3

First of all, I determine the cheating threshold - value of cheating costs beyond which there cannot be any incentives to cheat.

Lemma 1. *It is rational for player i to cheat IFF $d_i < F^{-1}\Big(\frac{1}{2} - v\Big)$ and she is indifferent between cheating or not if $d_i = F^{-1}\Big(\frac{1}{2} - v\Big)$.*

Proof. Note that $d_{-i} = -d_i$. Let player $-i$ be the underdog by the beginning of stage three (meaning that $d_{-i} < d_i$ and, therefore, $F(d_{-i}) < \frac{1}{2}$) . I compare her outcomes of cheating and fair play. When the underdog does not cheat (note that $\tilde{d}_i = d_i$ in this case), her payoff in the period three is:

$$\pi^3_{-i} = wF(-d_i).$$

When the underdog cheats, she pays $v > 0$ and sets $\tilde{d}_i = 0$. Her payoff is then:

$$\pi^{3*}_{-i} = wF(0) - v.$$

Therefore, it is rational for the underdog to cheat if $\pi^{3*}_{-i} - \pi^3_{-i} > 0$ and she is indifferent to cheat or not if $\pi^{3*}_j - \pi^3_j = 0$. Therefore, the cheating condition is (with $w = 1$):

$$F(0) - F(-d_i) > v.$$

By symmetry $F(0) = \frac{1}{2}$, and condition becomes

$$\frac{1}{2} - F(-d_i) > v. \tag{1}$$

Knowing the fact that

$$\min_{d_i} F(-d_i) \to 0,$$

it is now possible to conclude that the underdog player could have incentives to cheat if $v < \frac{1}{2}$, and could not have any incentives to cheat if $v > \frac{1}{2}$.

Rearranging inequality (1) I obtain:

$$F(-d_i) \leq \frac{1}{2} - v$$

$$-d_i < F^{-1}\left(\frac{1}{2} - v\right). \tag{2}$$

□

Equation (2) describes lower cheating boundary. It is evident that $F^{-1}\left(\frac{1}{2} - v\right) < 0$ (see Figure 1). Let the lower cheating boundary $-d^* = F^{-1}\left(\frac{1}{2} - v\right)$ and the upper cheating boundary $d^* = -F^{-1}\left(\frac{1}{2} - v\right)$.

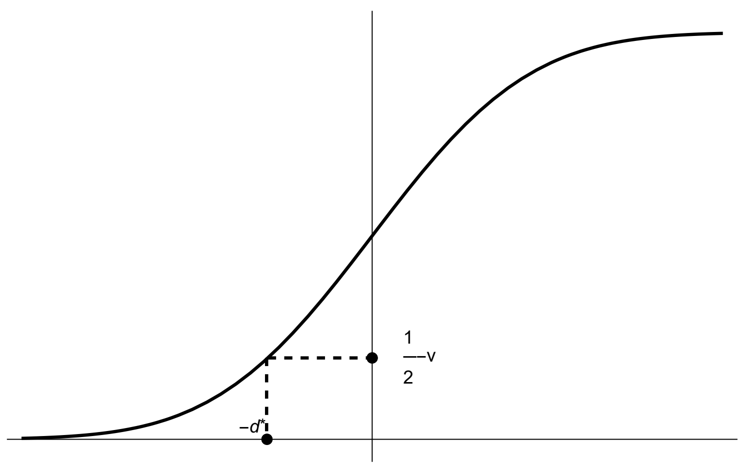

Figure 1: Cumulative distribution function of a symmetric distribution with lower cheating boundary $-d^*$

It is now evident that player i will cheat at the third stage whenever $d_i < -d^*$ and his opponent will cheat if $d_i > d^*$. Therefore, in order to sustain a fair play both d_j and d_k should $\in (-d^*, d^*)$.

It is also useful to investigate how cheating costs influence cheating boundary.

Proposition 1. *Cheating costs have strictly positive influence on the absolute value of cheating boundary:*

$$\frac{dd^*}{dv} > 0.$$

Proof. It is known that:

$$d^* = -F^{-1}\left(\frac{1}{2} - v\right)$$

and

$$v = \frac{1}{2} - F(-d^*).$$

Then:

$$\frac{dd^*}{dv} = \frac{1}{\frac{dv}{dd^*}} = \frac{1}{f(-d)},$$

which is strictly positive.

□

4.1.1 t=1

Now I consider the choice of the effort at time period 1 ($t = 1$). Assuming that player $-i$ is an underdog, interim case produces two different scenarios: **1)** If $d_i < d^*$ then there is no intention for underdog to cheat and **2)** f $d_i > d^*$ where underdog definitely cheats.

Scenario 1. $d_i < d^*$

To simplify notation, let $\Delta_j = x_j - x_k$ and $\Delta_k = x_k - x_j$. Note that $\Delta_j = -\Delta_k$.

Knowing that the cheating threshold is d^* we know that players end-up in the first scenario if:

$$-d^* < \Delta_i + \epsilon < d^*.$$

By rearranging the inequality I obtain:

$$-\Delta_i - d^* < \epsilon < -\Delta_i + d^*.$$

Corresponding probability to end-up in fair play scenario is:

$$\begin{aligned} &\mathsf{P}(-\Delta_i - d^* < \epsilon < -\Delta_i + d^*) = \\ &= F(-\Delta_i + d^*) - F(-\Delta_i - d^*). \end{aligned} \tag{3}$$

In fair play scenario player i wins the prize $w = 1$ if:

$$\Delta_i + \epsilon + \hat{\epsilon} > 0.$$

Therefore, player i's conditional probability of winning in fair play scenarios is:

$$\mathsf{P}(\Delta_i > -\epsilon - \hat{\epsilon} \mid -\Delta_i - d^* \leq \epsilon \leq -\Delta_i - d^*) =$$

$$= \frac{\mathsf{P}(\Delta_i > -\epsilon - \hat{\epsilon} \cap -\Delta_i - d^* \leq \epsilon \leq -\Delta_i - d^*)}{\mathsf{P}(-\Delta_i - d^* \leq \epsilon \leq -\Delta_i - d^*)} =$$

$$= \frac{\displaystyle\int_{-\Delta_i - d^*}^{-\Delta_i + d^*} F(\Delta_i + \epsilon) f(\epsilon) d\epsilon}{F(-\Delta_i + d^*) - F(-\Delta_i - d^*)}. \tag{4}$$

Putting together equations 3 and 4 I get player i's partial payoff for fair play scenario:

$$\int_{-\Delta_i - d^*}^{-\Delta_i + d^*} F(\Delta_i + \epsilon) f(\epsilon) d\epsilon = \int_{-d^*}^{d^*} F(\epsilon) f(\epsilon - \Delta_i) d\epsilon. \tag{5}$$

Note that $F(\cdot)$ is the CDF of $f(\epsilon)$. It is also known that $\mathbb{E}(\epsilon) = 0$.

Scenario 2. $d_i > d^*$

If $d_i > d^*$ then the underdog definitely cheats.

Player i has following probabilities in the cheating scenario:

$\mathsf{P}(\Delta_i + \epsilon > d^*) = F(\Delta_i - d^*)$ that she will be leading at $t = 3$ with corresponding payoff: $F(0) = \frac{1}{2}$;

$\mathsf{P}(\Delta_i + \epsilon < -d^*) = F(-\Delta_i - d^*)$ that she will be an underdog at $t = 3$ with corresponding payoff: $F(0) - v = \frac{1}{2} - v$.

Therefore, if player i ends up in a cheating scenario her partial payoff is:

$$\frac{1}{2} F(\Delta_i - d^*) + F(-\Delta_i - d^*)\Big(\frac{1}{2} - v\Big). \tag{6}$$

Payoffs

Putting together equations 5 and 6 it is possible to write down general for player i at $t = 1$:

$$\pi_i^1 = \int_{-d^*}^{d^*} F(\epsilon) f(\epsilon - \Delta_i) d\epsilon + \frac{1}{2} F(\Delta_i - d^*) + F(-\Delta_i - d^*)\Big(\frac{1}{2} - v\Big) - c(x_i).$$

FOC's for the problem above have the following form:

$$-\int_{-d^*}^{d^*} F(\epsilon) f'(\epsilon - \Delta_i) d\epsilon + \frac{1}{2} f(\Delta_i - d^*) - f(-\Delta_i - d^*)\Big(\frac{1}{2} - v\Big) = c'(x_i). \tag{7}$$

A sufficient condition for existence of a pure-strategy Nash equilibrium for the system above is:

$$\max_{x_i}\bigg(\int_{-d^*}^{d^*} F(\epsilon)f''(\epsilon-\Delta_i)d\epsilon+\frac{1}{2}f'(\Delta_i-d^*)$$
$$+f'(-\Delta_i-d^*)\Big(\frac{1}{2}-v\Big)\bigg)<\min_{x_i} c''(x_i). \quad (8)$$

The uniqueness of the solution is also supported by assumption that $c'(x_i) > 0$, $c''(x_i) > 0$ for all $x_i > 0$, and $c'(0) = 0$, which was already stated above.

However, due to the complexity of reaction functions (which satisfy first order condition (7)), it is difficult to characterise all conditions that ensure the existence of unique equilibrium. Some other necessary conditions will be discussed later on in the paper.

Assuming that the problem has a unique pure-strategy Nash equilibrium it is possible to solve it.

Lemma 2. *Assuming that there exists a symmetric pure-strategy equilibrium, equilibrium effort levels for* $v \in [0, \frac{1}{2}]$ *will have the following form:*

$$x_i = c'^{-1}\Big(-vf(d^*)+\int_{-d*}^{d^*}(f(\epsilon))^2 d\epsilon\Big).$$

Proof. The symmetry of players implies that unique pure-strategy Nash equilibrium is given by:

$$x_i = x_j = x_k. \quad (9)$$

Substituting (9) into FOC (7), I obtain:

$$-\int_{-d^*}^{d^*} F(\epsilon)f'(\epsilon)d\epsilon+\frac{1}{2}f(d^*)-f(-d^*)\Big(\frac{1}{2}-v\Big)=c'(x_i).$$

Rearranging and using the fact that $f(d^*) = f(-d^*)$ (by symmetry):

$$\underbrace{-\int_{-d^*}^{d^*} F(\epsilon)f'(\epsilon)d\epsilon}_{1}+vf(d^*)=c'(x_i). \quad (10)$$

I estimate the integral separately:

$$1) -\int_{-d^*}^{d^*} F(\epsilon)f'(\epsilon)d\epsilon =$$

Integration by parts yields:

$$du = f'(\epsilon)d\epsilon \qquad u = f(\epsilon)$$
$$v = F(\epsilon) \qquad dv = f(\epsilon)d\epsilon$$

$$= (F(-d^*) - F(d^*))f(d^*) + \int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon.$$

Using (2) implies:

$$= -2vf(d^*) + \int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon. \tag{11}$$

Substituting equation (11) into (10) I obtain:

$$-vf(d^*) + \int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon = c'(x_i). \tag{12}$$

Now it is possible to obtain the analytical form of the solution, which is:

$$x_i = c'^{-1}\Big(-vf(d^*) + \int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon \Big). \tag{13}$$

Where $d^* = -F^{-1}\Big(\frac{1}{2} - v\Big)$.

□

Lemma 2 implies that equilibrium solution, if there is one, is $x_k = x_j$. The problem is that even if $x_j = x_i^*$ is a consistent reply to $x_k = x_i^*$, it may not be the best one. Figure 2 demonstrates the situation in which it is indeed the best reply, while Figure 3 - the situation in which it is not.

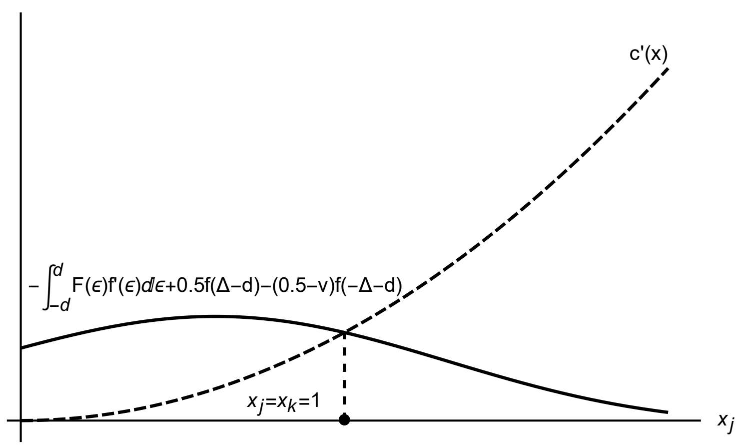

Figure 2: Sufficiently high variance results in $x_j = x_i^*$ being the best reply to $x_k = x_i^*$

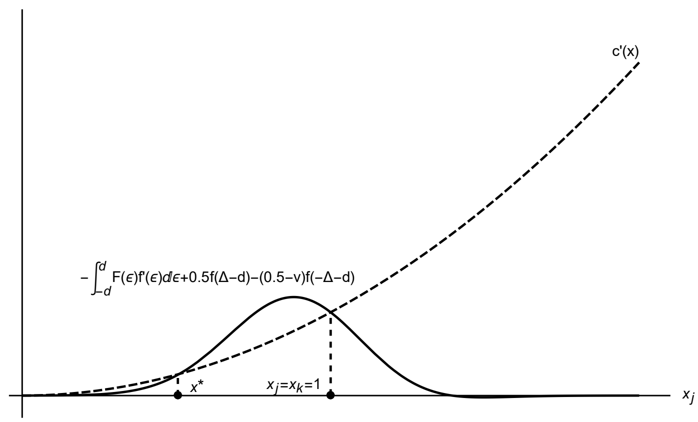

Figure 3: Low variance results in $x_j = x_i$ is not being the best reply to $x_k = x_i$ $(x_i^* > x^*)$

Evidently, the existence of unique equilibrium solution relies on the variance of shock distribution. The distribution function that has been used in a case demonstrated in Figure 2 has much higher standard deviation than the function used for the case in Figure 3.

Therefore, sufficiently large variance is an additional condition for existence of the unique equilibrium. Figure 2 demonstrates continuous well-behaved reaction functions corresponding to the case with high variance. Reaction functions for small variance case should be discontinuous as inflection points in Figure 3 imply that there always exists a value of x_k at which there are two equally good replies by player j and vice verse (the figure is not presented in the paper).

Another issue that may lead to discontinuous reaction function is non-zero mean of the shock distribution. However, it is not the case in a model proposed and will not be discussed in the paper.

It is also interesting to study how cheating costs influence players' effort levels. It is necessary just to investigate the sign of:

$$\frac{\partial}{\partial v}\Bigg(-vf(d^*) + \int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon \Bigg),$$

as it was assumed before that $c'(x) > 0$ and $c''(x) > 0$.

Let $F(\epsilon) = \int_0^{\epsilon} (f(t))^2 dt$ be the antiderivative of $(f(x))^2$ (such that $\frac{\partial F}{\partial \epsilon}$). Then:

$$\frac{d}{dv}\left(- vf(d^*) + \int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon \right) =$$

$$= -f(d^*) - vf'(d^*)\frac{dd^*}{dv} + ((f(d^*))^2 + (f(-d^*))^2)\frac{dd^*}{dv} =$$

$$= \underbrace{-f(d^*)}_{\mathbf{1}} - \underbrace{vf'(d^*)\frac{dd^*}{dv}}_{\mathbf{2}} + \underbrace{2(f(d^*))^2\frac{dd^*}{dv}}_{\mathbf{3}}.$$

It is evident that polynomial's second and third members are strictly positive as $f'(d^*) \leq 0$ for any unimodal symmetric distribution and it is known from the proposition 1 that $\frac{dd^*}{dv} > 0$.

However, first member is definitely negative. Therefore, the sign of the general form polynomial is ambiguous.

The sign of the $\frac{dx_i}{dv}$ is studied in further details in the next section for normal distribution special case.

5 Special cases: extreme values of v, and normally distributed shocks

It is interesting to observe the equilibrium effort levels of player at extreme values of cheating costs: $v = 0$ and $v = \frac{1}{2}$. Normal distributions assumption for shocks can also lead to valuable insights. For example, the assumption allows to investigate more deeply how equilibrium effort is influenced by cheating costs or shock distribution variance (uncertainty).

5.1 Negligible costs of cheating $v = 0$

Using Lemma 2, setting $v = 0$:

$$x_i = c'^{-1}\left(0f(d^*) + \int_0^0 (f(\epsilon))^2 d\epsilon\right) = 0.$$

Therefore, players maximise their payoffs when, x_j and x_k are set to 0. It is also obvious that if the cost of cheating is negligible, then the underdog player always chooses to cheat as it increases her chances to win.

To sum up, for $v = 0$ the unique equilibrium is for each player to set $x_i = 0$ and to cheat at $t = 3$ any time $d_i \neq 0$.

5.2 Significant costs of cheating $v = \frac{1}{2}$

When cheating costs are sufficiently high ($v = \frac{1}{2}$) no cheating occurs and agents just play a fair game. Using Lemma 2 and assuming some continuous symmetric distribution, I obtain:

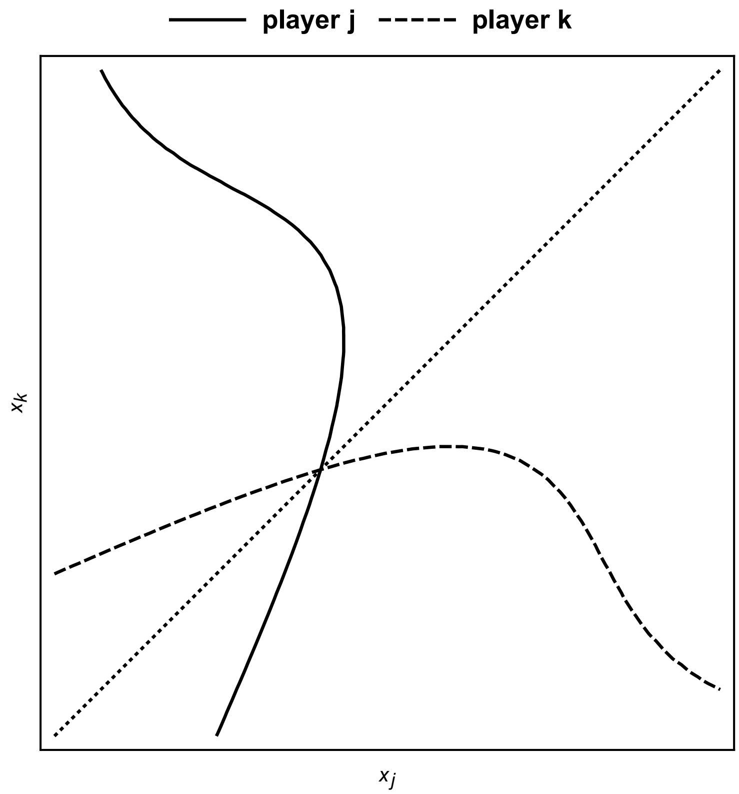

Figure 4: Well-behaved reaction functions corresponding to the shock that has distribution with sufficiently large variance

$$
\begin{aligned}
\mathrm{x}_i = c'^{-1}\Bigg(- vf(d^*) + \textstyle\int_{-d*}^{d^*} (f(\epsilon))^2 d\epsilon\Bigg) = \\
= c'^{-1}\Bigg(- vf(\infty) + \textstyle\int_{-\infty}^{\infty} (f(\epsilon))^2 d\epsilon\Bigg) = \\
= c'^{-1}\Bigg(\textstyle\int_{-\infty}^{\infty} (f(\epsilon))^2 d\epsilon\Bigg).
\end{aligned}
$$

Therefore, when v is too high neither player chooses to cheat at $t = 3$ and both of the them exert symmetric effort $x_i = c'^{-1}\Big(\int_{-\infty}^{\infty} (f(\epsilon))^2 d\epsilon\Big)$.

5.3 Normally distributed shocks

Assuming normally distributed shocks ($\epsilon \sim N(0, 2\sigma^2)$) and using Lemma 1 is now possible to derive the analytical form of the cheating boundary d_n^*.

Knowing that:

$$d^* = -F^{-1}\Big(\frac{1}{2} - v\Big),$$

the cheating boundary in the normal distribution case:

$$d_n^* = 2\sigma \operatorname{erf}^{-1}(2v). \tag{14}$$

Using Lemma 2 and specifically equation (13) it possible to obtain an equilibrium effort level in the normal distribution case.

$$x_i = c'^{-1}\bigg(- vf(d_n^*) + \int_{-d_n^*}^{d_n^*} (f(\epsilon))^2 d\epsilon \bigg) =$$
$$= c'^{-1}\bigg(- vf(d_n^*) + \frac{1}{2\sqrt{2\pi}\sigma}\Big(\operatorname{erf}\Big(\frac{d_n^*}{\sqrt{2}\sigma}\Big)\Big) \bigg) =$$
$$= c'^{-1}\bigg(- vf(d_n^*) + \frac{1}{2\sqrt{2\pi}\sigma}\Big(\operatorname{erf}\Big(\frac{d_n^*}{\sqrt{2}\sigma}\Big) + 1 - 1\Big) \bigg).$$

Therefore,

$$x_i = c'^{-1}\bigg(- vf(d_n^*) + \frac{1}{\sqrt{2\pi}\sigma}\Big(G(d_n^*) - \frac{1}{2}\Big) \bigg). \tag{15}$$

Where σ is a standard deviation and $G(\gamma)$ is a CDF of $g(\gamma)$, $E(\gamma^2) = \sigma^2$, $E(\gamma) = 0$.

It is now possible to investigate with more detail how cheating costs influence players' equilibrium effort levels.

Lemma 3. *In normal distribution case cheating costs have strictly positive influence on players' effort levels:*

$$\frac{\partial x_i}{\partial v} > 0.$$

Assuming quadratic costs ($c(x_i) = x_i^2$ and $c'(x_i) = x_i$), differentiate equation (15) w.r.t. v to obtain

$$\frac{\partial x_i}{\partial v} = -f(d_n^*) + e^{(\operatorname{erf}^{-1}(2v))^2}(g(d_n^*) - 2\sigma\sqrt{\pi}vf'(d_n^*)).$$

The sign is still not obvious. I expand PDF and CDF functions and simplify to obtain:

$$\frac{\partial x_i}{\partial v} = \frac{e^{-(\operatorname{erf}^{-1}(2v))^2}}{2\sigma\sqrt{\pi}} + \frac{v \operatorname{erf}^{-1}(2v)}{\sigma}. \tag{16}$$

It is evident that derivative both terms in (16) are strictly positive and, therefore:

$$\frac{\partial x_i}{\partial v} > 0.$$

It is also useful to analyse how uncertainty influences choice of the effort.

Lemma 4. *Uncertainty has a strictly negative influence on players' effort levels,* $\frac{\partial x_i}{\partial \sigma} < 0$.

Proof. I assume quadratic costs ($c(x_i) = x_i^2$ and $c'(x_i) = x_i$)). Differentiate (15) w.r.t. σ to obtain

$$\frac{\partial x_i}{\partial \sigma} = \frac{v e^{-(\text{erf}^{-1}(2v))^2}}{2\sqrt{\pi}\sigma^2} - \frac{\text{erf}\left(\sqrt{2}\text{erf}^{-1}(2v)\right)}{2\sqrt{2\pi}\sigma^2}. \tag{17}$$

The sign of equation (17) is not obvious. Therefore, I use its graphical representation to investigate it (see Figure 5).

The Figure makes it evident that for and $v \in \left(0, \frac{1}{2}\right)$, $\frac{\partial x_i}{\partial \sigma}$ has a negative sign. □

It means that uncertainty has a negative influence on player's effort levels.

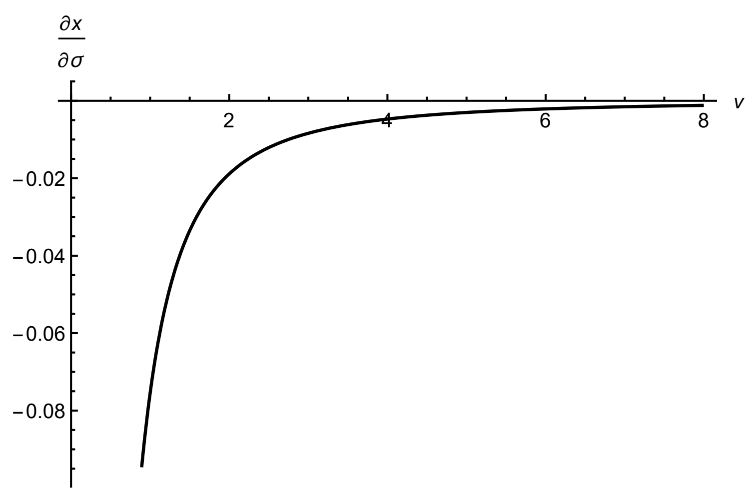

Figure 5: Derivative of the effort function w.r.t σ ($v = \frac{1}{4}$)

6 Limitations and further development

The model has several limitations, the solution to which can lead to a valuable insights:

1. The model assumes non-overlapping effort levels, which may not be the case in some real-world situations. For example, when one of the players cheats, she sets her rival's relative advantage (or her own disadvantage) to zero; however, if we assume that model describes cheating in the R&D race, it is unlikely that rivals' gathered knowledge (effort) is similar (it

will be different in terms of findings, results, approaches, etc.). It means that when an underdog player cheats and steals rival's information, her relative disadvantage, probably, will not be set to zero, but rather will turn into advantage as she will have both her own and rival's knowledge: $x_{cheat} = x_j + x_k$. Of course, it should also be interesting to research on what happens if we assume that some share of knowledge α indeed overlaps. It will make the model more realistic and possibly will provide some insights on how the knowledge overlap influences cheating decision and the choice of effort level;

2. It would be also interesting to investigate the setting with shocks that are not i.i.d.. The modification is reasonable as in real life situation the noise of monitoring cheating and players' level is rather heterogeneous. This setting could lead to cases, where the impact of the cheating costs on effort is ambiguous (could be both negative and positive);

3. It is not always the case that competing players reside in the same environment with similar cheating costs. The situation in which players have asymmetric cheating costs is also worth exploring. For example, companies that compete on the international market, but reside in different countries with different penalties for cheating $v_j \neq v_k$. An appropriate example is the cyber arms race between United States and China. This modification can lead to the development of cheating penalty regulation guidelines in the asymmetric environment (e.g. international competition);

4. The perfect information assumption could also be misleading. Incomplete information game variations are also worth exploring. It can also make model more real and therefore, more applicable to the real world situations.

5. Unique equilibrium conditions are very hard to characterize for this model. However, researching for analytical set of unique equilibrium conditions might also be beneficial.

Mentioned modifications can change the baseline model results.

References

Reuters. Update 1-u.s. republican senator seeks briefings on reported china hacking attack. *www.cnbc.com*, Oct 2018.

Edward P. Lazear and Sherwin Rosen. Rank-Order Tournaments as Optimum Labor Contracts. *Journal of Political Economy*, 1981. ISSN 0022-3808. doi: 10.1086/261010.

Barry J . Nalebuff and Joseph E. Stiglitz. Prizes and Incentives : Towards a General Theory of Compensation and Competition Author (s): Reviewed work (s): Published by : RAND Corporation. *The Bell Journal of Economics*, 14(1):21–43, 1983.

Oliver D Hart. The Market Mechanism as an Incentive Scheme. *Source: The Bell Journal of Economics*, 1983. ISSN 0361915X. doi: 10.2307/3003639.

Bengt Holmstrom. Moral Hazard in Teams Author (s): Bengt Holmstrom Published by : RAND Corporation Stable URL : http://www.jstor.org/stable/3003457 Moral hazard in teams. *The Bell Journal of Economics*, 13(2):324–340, 1982.

Jerry R Green and Nancy L Stokey. A Comparison of Tournaments and Contracts Author (s): Jerry R . Green and Nancy L . Stokey Source : Journal of Political Economy , Vol . 91 , No . 3 (Jun ., 1983), pp . 349-364 Published by : The University of Chicago Press Stable URL : http://www.jst. 91(3): 349–364, 2016.

Mary O'Keeffe, W Kip Viscusi, and Richard J Zeckhauser. Economic Contests : Comparative Reward Schemes. *Journal of Labor Economics*, 1984. ISSN 0734-306X. doi: 10.1086/298022.

Aleksander Berentsen and Yvan Lengwiler. Fraudulent Accounting and Other Doping Games. 2005.

Matthias Kräkel. Doping and cheating in contest-like situations. *European Journal of Political Economy*, 2007. ISSN 01762680. doi: 10.1016/j.ejpoleco.2006.11.003.

Philip A Curry and Steeve Mongrain. Deterrence in Rank - Order Tournaments. 2000.

Scott M. Gilpatric. Cheating in contests. *Economic Inquiry*, 2011. ISSN 00952583. doi: 10.1111/j.1465-7295.2010.00244.x.

C. Jill Stowe and Scott M. Gilpatric. Cheating and Enforcement in Asymmetric Rank-Order Tournaments. *Southern Economic Journal*, 77 (1):1–14, 2010. ISSN 0038-4038. doi: 10.4284/sej.2010.77.1.1. URL `http://doi.wiley.com/10.4284/sej.2010.77.1.1`.

Philipp Denter and Dana Sisak. Do polls create momentum in political competition? *Journal of Public Economics*, 130:1–14, 2015. ISSN 00472727. doi: 10.1016/j.jpubeco.2015.07.003. URL http://dx.doi.org/10.1016/j.jpubeco.2015.07.003.

Philipp Denter and Dana Sisak. Head starts in dynamic tournaments? *Economics Letters*, 149:94–97, 2016. ISSN 01651765. doi: 10.1016/j.econlet.2016.10.008. URL http://dx.doi.org/10.1016/j.econlet.2016.10.008.

Ron Siegel. Asymmetric contests with head starts and nonmonotonic costs. *American Economic Journal: Microeconomics*, 6(3):59–105, 2014. ISSN 19457685. doi: 10.1257/mic.6.3.59.